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UNIVERSITÉ DE MONTRÉAL

# OPTIMAL OPERATION OF A MULTI-PURPOSE RESERVOIR

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MÉMOIRE PRÉSENTÉ EN VUE DE L'OBTENTION
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Ce mémoire intitulé :

OPTIMAL OPERATION OF A
MULTI-PURPOSE RESERVOIR

présenté par : MAHVASH MOHAMMADI Batoul
en vue de l'obtention du diplôme de : Maîtrise ès sciences appliquées
a été dûment accepté par le jury d'examen constitué de :

M. SOUMIS François, Ph.D., président
M. TURGEON André, Ph.D., membre et directeur de recherche
M. LEFEBVRE Mario, Ph.D., membre

*Dedicated to my parents*

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# RÉSUMÉ

L'objectif de cette thèse est de déterminer la règle optimale de gestion d'un réservoir qui doit satisfaire les besoins conflictuels en eau d'un grand nombre d'utilisateurs.Un des objectifs est d'alimenter à l'année longue la centrale hydroélectrique en aval du réservoir. Un autre est de maintenir le niveau du réservoir suffisamment élevé durant l'été pour permettre la navigation de plaisance et d'autres activités récréatives. Un troisième objectif, et non le moindre, est de minimiser les risques d'inondation.

Il existe plusieurs façons de modéliser un problème d'optimisation multicritères. La plus répandue est la méthode des contraintes qui consiste à ajouter suffisamment de contraintes au modèle d'optimisation mathématique pour que la solution obtenue respecte les différents critères. Cette approche est valide pour un problème déterministe mais pas pour un problème stochastique. Dans les problèmes stochastiques, il y a toujours le risque qu'un ou plusieurs objectifs ne soient pas satisfaits. La façon la plus facile de résoudre ce problème est d'ajouter des fonctions de pénalité au modèle d'optimisation qui pénalisent le non-respect des objectifs. C'est l'approche que nous avons suivie dans cette thèse. Nous avons construit une fonction objectif linéaire par parties et concave pour le modèle d'optimisation stochastique qui donne les résultats désirés.

Différentes méthodes d'optimisation peuvent être utilisées pour résoudre les problèmes de gestion des réservoirs. Le choix de la méthode dépend de la nature et de la taille du problème. Plus précisément, selon que le problème est linéaire ou non linéaire, déterministe ou stochastique, convexe ou non convexe, séparable ou non séparable, le choix de la méthode peut être différent. Les méthodes d'optimisation les plus utilisées sont la programmation linéaire, la programmation non linéaire, la programmation dynamique et les méta-heuristiques.

La programmation dynamique (DP) est utilisée dans cette thèse pour résoudre le problème de gestion du réservoir. Cette méthode n'est pas une méthode d'optimisation mais bien une méthode de décomposition. Elle décompose un problème dynamique comprenant plusieurs étapes en une série de problèmes d'une étape. La très grande popularité de cette méthode est due au fait qu'elle donne une solution de type feedback et qu'elle résout des problèmes d'optimisation stochastique. Malheureusement, DP est incapable de résoudre des problèmes de grande taille. Étant donné qu'elle donne une solution feedback, le temps de calcul et la mémoire requise pour stocker la solution augmentent exponentiellement avec le nombre de variables d'état. Il s'en suit que seuls les problèmes de moins de cinq variables d'état peuvent être résolus dans un temps raisonnable avec la programmation dynamique.

Le problème d'optimiser la règle de gestion d'un réservoir est résolu dans ce thèse pour des apports d'eau au réservoir représentés par des distributions de probabilité de même que par des ensembles de scénarios. Dans le premier cas, le problème est résolu avec la programmation dynamique stochastique et, dans le second, avec la méthode de « Sampling Stochastic Dynamic Programming (SSDP) ».

Les distributions de probabilité des apports d'eau journaliers au réservoir ont été déterminées à partir de l'historique de 84 années d'apports pour ce réservoir. Ces données ont du tout d'abord être normalisées étant donné que les apports sont très asymétriques sur une base de temps journalière. Des modèles autorégressifs d'ordre un, et donc markoviens, ont été construits par la suite pour représenter les apports pour chacun des 365 jours de l'année. Ces modèles ont été utilisés pour déterminer les distributions de probabilité des apports et générer des scénarios d'apports synthétiques.

La méthode SSDP a été appliquée au problème de gestion du réservoir en supposant tout d'abord que les probabilités de transition d'un scénario d'apport à un autre scénario à la fin de chaque journée étaient celles données par le modèle autorégressif. Le problème a été résolu par la suite en supposant que les probabilités de transition étaient toutes égales, ce qui a donné de moins bons résultats. Trois ensembles de scénarios d'apports différents ont été utilisés pour résoudre les problèmes. Le premier contenait 100 scénarios, le deuxième 200 et le troisième 300. Le but était de déterminer si la solution était sensible au nombre de scénarios utilisés. Ce ne fut pas le cas. Pour le problème d'optimisation résolu avec la programmation dynamique stochastique, seulement onze apports différents ont été utilisés, comparativement aux 100, 200 et 300 apports utilisés dans SSDP. Les résultats obtenus avec la programmation dynamique sont pourtant aussi bons, sinon meilleurs, que ceux obtenus avec SSDP. Nous sommes arrivés à la conclusion qu'il n'y a pas de réels avantages à utiliser la méthode SSDP de préférence à la programmation dynamique pour déterminer la règle de gestion d'un réservoir.

## ABSTRACT

The objective of this thesis is to determine the optimal operating policy of a reservoir that satisfies several conflicting objectives. One of these objectives is to continuously feed water to a downstream hydroelectric powerplant. Another is to maintain the reservoir level high enough during summer to permit navigation and other recreational activities. A third objective, and not the least important, is to minimize the risks of flooding.

A multi-objective optimization problem can be dealt with in many different ways. The easiest way is to add constraints to the mathematical optimization problem that remove solutions that do not respect all objectives. This method is valid for deterministic optimization problems but not for stochastic ones. The risk of not satisfying some objectives always exists in a stochastic optimization problem, so that the objectives will not always be met. The easiest way of solving the problem is to add penalty functions to the optimization problem so as penalize the non-respect of the objectives. This approach was followed in this thesis. We built a concave piecewise linear penalty function that gives the desired results.

Several optimization techniques may be used to solve reservoir management problems. The choice of the technique depends of the size of the problem and on whether the problem is linear or nonlinear, deterministic or stochastic, convex or non-convex, and

separable or not separable. The most popular optimization techniques are Linear Programming, Nonlinear Programming, Dynamic Programming, and the metaheuristic procedures.

Dynamic Programming (DP) is the technique used in this thesis to solve the reservoir management problem. This technique is not an optimization technique but a decomposition technique. It decomposes a multi-stage optimization problem into a series of one-stage problems. The popularity and success of this method is due to the fact that it gives feedback solutions and solves stochastic optimization problems. Unfortunately, DP cannot solve large optimization problems because the computational and memory requirements grow exponentially with the number of state variables. Bellman called this the «curse of dimensionality». Because of it, DP cannot generally be used for dynamic optimization problems that have more than four state variables.

The reservoir management problem was solved in this thesis with the probability distributions of the reservoir inflows and a set of inflow scenarios. The problem was solved with SDP when probabilities are used and with Sampling Stochastic Dynamic Programming (SSDP) when scenarios are used.

The probability distributions of the daily reservoir inflows were determined using the historical inflow record of 84 years. Since these inflows are usually skewed, the historical data were first normalized. Lag-one autoregressive models of the daily inflows were built afterwards. These models were used to determine the conditional probability distributions of the inflows and generate synthetic inflow scenarios.

The reservoir management problem was solved twice with SSDP. First, the transition probability from one scenario to another at the end of a day was set equal to the conditional probability determined with the autoregressive model. Next, the transition probabilities were all set to the same value, which did not give very good results. Three set of inflow scenarios were used to solve the optimization problems. The first had 100 scenarios, the second 200 and the third 300. The aim was to determine if the reservoir operating policy obtained changes with the number of scenarios used to solve the problem. It did not significantly change. For the problem solved with Stochastic Dynamic Programming, only eleven inflow values were used to solve the problem, compared to 100, 200 and 300 values with SSDP. The results obtained with SDP are unexpectedly as good, if not better, than those obtained with SSDP. We come to the conclusion that there are no real advantages in using SSDP instead of SDP to determine the operating policy of a reservoir

# CONDENSÉ EN FRANÇAIS

L'objectif de cette thèse est de déterminer la règle optimale de gestion d'un réservoir qui doit satisfaire les besoins conflictuels en eau d'un grand nombre d'utilisateurs. Un des objectifs est d'alimenter à l'année longue la centrale hydroélectrique en aval du réservoir. Un autre est de maintenir le niveau du réservoir suffisamment élevé durant l'été pour permettre la navigation de plaisance et d'autres activités récréatives. Un troisième objectif, et non le moindre, est de minimiser les risques d'inondation.

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Différentes méthodes d'optimisation peuvent être utilisées pour résoudre les problèmes de gestion des réservoirs. Le choix de la méthode dépend de la nature et de la taille du problème. Plus précisément, selon que le problème est linéaire ou non linéaire, déterministe ou stochastique, convexe ou non convexe, séparable ou non séparable, le choix de la méthode peut être différent. Les méthodes d'optimisation les plus utilisées sont la programmation linéaire, la programmation non linéaire, la programmation dynamique et les méta-heuristiques.

La programmation dynamique (DP) est utilisée dans cette thèse pour résoudre le problème de gestion du réservoir. Cette méthode, développée par Bellman (1957) dans les années cinquante, n'est pas une méthode d'optimisation mais bien une méthode de décomposition. Elle décompose un problème dynamique comprenant plusieurs étapes en une série de problèmes d'une étape. La très grande popularité de cette méthode est due au fait qu'elle donne une solution de type feedback et qu'elle résout des problèmes d'optimisation stochastique. Malheureusement, DP est incapable de résoudre des problèmes de grande taille. Étant donné qu'elle donne une solution feedback, le temps de calcul et la mémoire requise pour stocker la solution augmentent exponentiellement avec le nombre de variables d'état. Il s'en suit que seuls les problèmes de moins de cinq variables d'état peuvent être résolus dans un temps raisonnable avec la programmation dynamique.

Le problème d'optimiser la règle de gestion d'un réservoir est résolu dans cette thèse pour des apports d'eau au réservoir représentés par des distributions de probabilité de même que par des ensembles de scénarios. Dans le premier cas, le problème est résolu avec la programmation dynamique stochastique et, dans le second, avec la méthode de « Sampling Stochastic Dynamic Programming (SSDP) » développée par Kelman et al. (1990).

Les distributions de probabilité des apports d'eau journaliers au réservoir ont été déterminées à partir de l'historique de 84 années d'apports au réservoir. Ces données ont du tout d'abord être normalisées étant donné que les apports sont très asymétriques sur une base de temps journalière. Des modèles autorégressifs d'ordre un, et donc markoviens, ont été construits par la suite pour représenter les apports pour chacun des 365 jours de l'année. Ces modèles ont été utilisés pour déterminer les distributions de probabilité des apports et générer des scénarios d'apports synthétiques.

La méthode SSDP a été appliquée au problème de gestion du réservoir en supposant tout d'abord que les probabilités de transition d'un scénario d'apport à un autre scénario à la fin de chaque journée étaient celles données par le modèle autorégressif. Le problème a été résolu par la suite en supposant que les probabilités de transition étaient toutes égales, ce qui a donné de moins bons résultats. Trois ensembles de scénarios d'apports différents ont été utilisés pour résoudre les problèmes. Le premier contenait 100 scénarios, le deuxième 200 et le troisième 300. Le but était de déterminer si la solution était sensible au nombre de scénarios utilisés. Ce ne fut pas le cas. Pour le problème d'optimisation résolu avec la programmation dynamique stochastique, seulement onze apports différents ont été utilisés, comparativement aux 100, 200 et 300 apports utilisés dans SSDP. Les résultats obtenus avec la programmation dynamique sont pourtant aussi bons, sinon meilleurs, que ceux obtenus avec SSDP. Nous sommes arrivés à la conclusion qu'il n'y a pas de réels avantages à utiliser la méthode SSDP de préférence à la programmation dynamique pour déterminer la règle de gestion d'un réservoir.

Le problème de déterminer la politique optimale de gestion du réservoir peut être formulé mathématiquement comme suit :

Maximiser $\sum_{t=1}^{T} b(R_t) + \phi(S_{T+1})$

sous :

$$S_{t+1} = S_t + (Q_t - R_t) \cdot c \quad ; \quad t = 1,2,\ldots,T \quad S_1 \text{ connu,}$$

$$S_{t+1}^{\min} \leq S_{t+1} \leq S_{t+1}^{\max} \quad \text{et} \quad R_t \geq 0 \quad ; \quad t = 1,2,\ldots,T$$

où $c$ est le facteur de conversion des $(m^3/s) \times jour$ en $hm^3$, $S_t$, le contenu du réservoir en $hm^3$ au début du jour $t$, $Q_t$, l'apport au réservoir en $m^3/s$ dans le jour $t$, et où $R_t$ est le soutirage du réservoir en $m^3/s$ au jour $t$. La fonction $b(R_t)$ donne le bénéfice d'un soutirage de $R_t$ $m^3/s$ tandis que la fonction $\phi(S_{T+1})$ donne la valeur de l'eau stockée dans le réservoir à la fin de l'horizon.

Lorsque les apports $Q_1, Q_2, ..., Q_T$ sont connus au départ, la règle optimale de gestion du réservoir peut être déterminée en solutionnant à rebours la fonctionnelle :

$$f_t(S_t) = \max_{R_t}[b(R_t) + f_{t+1}(S_{t+1})]$$

en commençant au temps $T$ avec $f_{T+1}(S_{T+1}) = \phi(S_{T+1})$. Dans ce problème, le soutirage $R_t$ est la variable de décision. La valeur du soutirage doit naturellement respecter l'équation d'état du réservoir de même que les bornes sur les variables.

La programmation dynamique stochastique est utilisée quand les apports d'eau au réservoir ne sont pas connus d'avance mais que les distributions de probabilité de ces apports sont connues. Le problème devient dans ce cas à maximiser l'espérance mathématique des profits. Mathématiquement, le problème consiste à résoudre à rebours la fonctionnelle suivante :

$$f_t(S_t) = \max_{R_t} \underset{Q_t}{E}\{b(R_t) + f_{t+1}(S_{t+1})\}$$

Dans la pratique, l'apport au réservoir est habituellement connu quand le soutirage est fixé de sorte que la fonctionnelle peut s'écrire :

$$f_t(S_t) = \underset{Q_t}{E}\left[\max_{R_t}\{B_t(S_{t+1}, R_t) + f_{t+1}(S_{t+1})\}\right] .$$

Puisque le symbole $\underset{Q_t}{E}$, qui désigne l'espérance mathématique, est situé avant le "max" dans la dernière équation, l'apport $Q_t$ est donc connu quand la valeur de $R_t$ est choisie.

L'apport au réservoir à l'étape $t$ est presque toujours corrélé à celui de l'étape $t-1$ quand l'étape est courte, comme le jour par exemple. Il arrive même que l'apport soit non seulement corrélé à celui de l'étape précédente mais à ceux de plusieurs étapes précédentes. La corrélation entre les apports des étapes $t$ et $t-1$, appelée autocorrélation d'ordre un, peut être prise en considération dans la fonctionnelle de Bellman en ajoutant une nouvelle variable d'état $Q_{t-1}$ au problème. La fonctionnelle devient alors

$$f_t(S_t, Q_{t-1}) = \underset{Q_t|Q_{t-1}}{E}\left[\max_{R_t}\left\{B_t\left(S_{t+1}, R_t\right) + f_{t+1}(S_{t+1}, Q_t)\right\}\right]$$

où $\underset{Q_t|Q_{-1}}{E}$ désigne l'espérance mathématique conditionnelle.

Quand $Q_t$ est corrélé non seulement à $Q_{t-1}$, mais à $Q_{t-2}, Q_{t-3}, \ldots, Q_{t-n}$, l'équation récursive devient :

$$f_t(S_t, Q_{t-1}, \ldots, Q_{t-n-1}) = \underset{Q_t|Q_{t-1},\ldots,Q_{t-n-1}}{E}\left[\max_{R_t}\left\{B_t\left(S_{t+1}, R_t\right) + f_{t+1}(S_{t+1}, Q_t, \ldots, Q_{t-n})\right\}\right]$$

Cette équation ne pourra être résolue dans un temps de calcul raisonnable que si $n$ est plus petit que cinq.

Comme nous l'avons déjà mentionné, la méthode SSDP de Kelman et al. (1990) a été utilisée pour déterminer la règle optimale de gestion du réservoir à partir d'un ensemble de scénarios d'apports. Toutefois, étant donné que cette méthode utilise des probabilités de transition d'un scénario à un autre scénario à la fin de chaque étape, elle n'est pas tellement différente de la programmation dynamique stochastique qui elle aussi utilise des probabilités de transition. Le problème d'optimisation se résout de la façon suivante avec SSDP:

1) Pour chaque scénario $i$ $(i = 1,2..,I)$, résoudre le problème d'optimisation suivant à l'étape $t$:

$$\underset{R_r}{\text{Maximize}} \left\{ B_t\left(S_{t+1}, R_t\right) + \sum_{j=1}^{I} f_{t+1}\left(S_{t+1}, j\right) \cdot \Pr\left(j|i\right) \right\}$$

Sous :

$$S_{t+1} = S_t + \left(Q_t - R_t\right) \cdot c \qquad ; \quad t = 1,2,...,T \quad ; \quad S_1 \text{ connu,}$$

$$S_{t+1}^{\min} \leq S_{t+1} \leq S_{t+1}^{\max} \qquad \text{et} \quad R_t \geq 0$$

où $\Pr\left(j|i\right)$ désigne la probabilité que l'apport corresponde à celui du scénario $j$ à l'étape $t$ étant donné qu'il correspondait à celui du scénario $i$ à l'étape $t-1$. Désigner par $R_t^i$ la solution du problème.

2) Poser $f_t\left(S_t, i\right) = B_t\left(S_{t+1}, R_t^i\right) + f_{t+1}\left(S_{t+1}, i\right)$.

Dans cette formulation de SSDP, la variable hydrologique est $i$, le numéro du scénario d'apport. Un choix simple pour $\Pr\left(j|i\right)$ est de fixer $\Pr\left(j|i\right) = 1$ pour $j = i$ et $\Pr\left(j|i\right) = 0$ sinon. Ce choix est équivalent à exécuter une optimisation déterministe avec le scénario $i$. Un autre choix simple est de supposer que $\Pr\left(j|i\right)$ a la même valeur pour tous les $j$ et $i$. Autrement dit, $\Pr(j|i) = 1/I \quad \forall j,i$. Le choix le plus judicieux est naturellement de déterminer la valeur réelle de $\Pr\left(j|i\right)$, c'est-à-dire celle qui reflète la distribution de probabilité des apports à l'étape $t$. Il existe deux façons de déterminer cette probabilité. La première, dite non paramétrique, consiste à calculer la probabilité à partir de l'historique des apports. La deuxième, nommée paramétrique, demande de construire un modèle mathématique des apports.

La méthode non paramétrique est difficilement utilisable en hydrologie parce que l'on a toujours très peu de données. Par exemple, si l'historique est de 50 ans, ce qui est déjà passablement long, ceci signifie que l'on dispose de 50 données pour l'étape $t$. Ce nombre est insuffisant pour déterminer une probabilité conditionnelle.

Les méthodes paramétriques les plus utilisées demandent de construire des modèles autorégressifs ou des modèles autorégressifs à moyennes mobiles des apports (Salas, 1980). Par exemple, désignons par $Q_t$ la variable aléatoire représentant l'apport au réservoir dans la période $t$ et supposons que cette variable a une distribution de probabilité de type gaussien dont la moyenne est égale à $\mu_t$. Fixons $Z_t = Q_t - \mu_t$ et supposons que $Z_t$ est linéairement corrélé à $Z_{t-1}, Z_{t-2}, ..., Z_{t-p}$. La relation entre $Z_t$ et les autres variables peut dans ce cas s'écrire comme suit :

$$Z_t = \phi_{t,1} Z_{t-1} + \phi_{t,2} Z_{t-2} + ... + \phi_{t,p} Z_{t-p} + \varepsilon_t$$

où $\phi_{t,1}, \phi_{t,2}, ..., \phi_{t,p}$ sont des coefficients d'autoregression et $\varepsilon_t$ une variable aléatoire normalement distribuée et indépendante dont la moyenne est égale à zéro. Cette variable représente en fait l'erreur d'estimation de $Z_t$, connaissant $Z_{t-1}, Z_{t-2}, ..., Z_{t-p}$. Si l'erreur est toujours nulle, ceci signifie que $Z_t$, et donc $Q_t$, n'est pas aléatoire puisque, connaissant $Z_{t-1}, Z_{t-2}, ..., Z_{t-p}$, on peut déterminer exactement sa valeur. Plus l'erreur est petite, et plus exactement la variance de l'erreur, plus la corrélation entre $Z_t$ et $Z_{t-1}, Z_{t-2}, ..., Z_{t-p}$ est grande. Les coefficients d'autoregression peuvent être déterminés par les méthodes de moments, de moindres carrés ou de maximum de vraisemblance.

La variable aléatoire $Z_t$ pourrait probablement être aussi représentée par le modèle autorégressif à moyenne mobile suivant:

$$Z_t = \varepsilon_t - \sum_{j=0}^{q} \theta_{t,j} \varepsilon_{t-j}$$

Une autre possibilité serait d'utiliser un modèle mixte de type ARMA comme le suivant :

$$Z_t = \phi_{t,1} Z_{t-1} + ... + \phi_{t,p} Z_{t-p} + \varepsilon_t - \theta_{t,1} \varepsilon_{t-1} - ... - \theta_{t,q} \varepsilon_{t-q}$$

Il fut supposé ci-dessus que la variable aléatoire $Q_t$ représentant l'apport au réservoir dans la période $t$, avait une distribution de probabilité gaussienne. La raison est toute simple. La somme de plusieurs variables de distribution gaussienne est une gaussienne. Par conséquent, si les variables $Z_{t-1}, Z_{t-2}, ..., Z_{t-p}$ dans les équations de ci-dessus ont une distribution gaussienne, $Z_t$ a une distribution gaussienne. Ceci ne s'applique malheureusement pas aux autres distributions de probabilité.

En pratique la variable $Q_t$ est rarement gaussienne. Plus le pas de temps est court, plus la distribution de probabilité est asymétrique. Salas (1980) conseille fortement de ne pas utiliser de distributions asymétriques dans les modèles autorégressifs mais d'utiliser plutôt des transformations mathématiques de façon à obtenir des variables aléatoires normalement distribuées. Dans cette thèse, nous avons utilisé des transformations de type log-normal à deux et trois paramètres et de type gamma pour obtenir des données normalement distribuées. Le choix de la transformation a été fait avec le test de Filliben.

Bref les probabilités conditionnelles, $\Pr(j|i)$, utilisées pour résoudre le problème de gestion du réservoir avec la méthode SSDP ont été déterminées avec un modèle autorégressif construit avec des données transformées de distribution gaussienne. Les différents modèles d'optimisation ont été programmés dans le langage C++ et ont été testés avec les 84 années d'apports de l'historique de même qu'aves les 100, 200 et 300 scénarios d'apports synthétiques. Les résultats obtenus sont présentés dans la thèse.

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## LIST OF SYMBOLS

| | |
|---|---|
| AR ( $p$ ) | Autoregressive model of order $p$ |
| ARMA | Autoregressive moving average |
| $b(R_t)$ | Benefit for releasing of $R_t$ $m^3/s$ from the reservoir on day $t$ |
| DDDP | Discrete Differential Dynamic Programming |
| DP | Dynamic Programming |
| DPSA | Dynamic Programming Successive Approximations |
| $\underset{Q_t}{E}$ | Expected value with respect to variable $Q_t$ |
| $\underset{Q_t \mid Q_{t-1}}{E}$ | Conditional expectation |
| $f(X_1, X_2, \ldots)$ | Joint distribution |
| g | Sample skewness coefficient |
| GRG | Generalized Reduced Gradient method |
| IDP | Incremental Dynamic Programming |
| ISO | Implicit Stochastic Optimization |
| LP | Linear Programming |
| MA | Moving average |
| MOM | Method of Multipliers |
| NLP | Nonlinear Programming |

| | | |
|---|---|---|
| NWS | National Weather Service | |
| $P(S_t)$ | Penalty function | |
| $\Pr(j \mid i)$ | Conditional probability of scenario $j$ given scenario $i$. | |
| $Q_t$ | Reservoir inflow on day $t$. | $m^3/s$ |
| $Q_{k,i}$ | Registered inflow on day $k$ of year $i$ | |
| $R_t$ | Reservoir discharge on day $t$. | $m^3/s$ |
| S | Standard deviation | |
| $S_t$ | Reservoir content at the beginning of day $t$ | $hm^3$ |
| $S^2$ | Sample variance | |
| SDP | Stochastic Dynamic Programming | |
| SLP | Sequential Linear Programming | |
| SQP | Sequential Quadratic Programming | |
| $\{X_t\}_{t\in I}$ | Time series | |
| $\overline{X}$ | Sample mean | |
| $\Gamma(.)$ | Gamma function | |
| $\gamma_k$ | Autocovariance at lag $k$ | |
| $\mu_t$ | Expected value of stochastic process | |

| | |
|---|---|
| $\rho_k$ | Autocorrelation coefficient at lag $k$ |
| $\sigma_t^2$ | Variance of stochastic variable |
| $\phi(S_{T+1})$ | Value of the water stored in the reservoir at the end of the time horizon |

# INTRODUCTION

In most countries of the world, there are rivers on which dams and reservoirs were built to control the stream flow in order to achieve some set of specified objectives like preventing flooding, supplying water to municipalities, producing hydroelectric energy, allowing navigation, flood control operations, water supply operation, etc. This multi-objective optimization problem is usually difficult to solve because the water inflow to the river is stochastic and cannot be forecasted long in advance.

The quantity of water available varies from year to year and within a year. There may be many consecutive years of high inflows followed by many years of low inflows. There may also be many consecutive days of high inflows and low inflows within a year. Managing the reservoirs to prevent flooding and shortages of water may be difficult during the long periods of high and low inflows.

In Quebec, the reservoir inflows are very low during winter because the precipitation is in the form which does not melt before spring. The demand for electricity is high however because a large percentage of the houses and buildings are heated with electricity. The only way to satisfy the winter electricity demand is to build large reservoirs and fill them before winter begins. Fortunately, the inflows are very high during the spring thaw. In fact, more than fifty percent of the total annual inflow occurs in this season. The excess water of this season is stored in the reservoirs for the next winter.

The reservoirs must be managed to meet the different objectives in the best possible way. Some of these objectives may be conflicting. For instance, the objectives of minimizing the risk of flooding and maximizing the generation of electricity are conflicting. The risk of flooding decreases when the reservoir level decreases whereas the electricity generation increases when the reservoir level increases. Finding the best reservoir level may in this case be a difficult problem to solve. A similar conflict occurs with the objective of maintaining the reservoir level high during the summer to permit navigation and other recreational activities. Maintaining the reservoir level high may increase the probability of flooding. Determining the correct level may, in this case, be a political problem.

The problem of finding the optimal operating policy of a reservoir is also difficult to solve because the inflows are stochastic and cannot be forecasted long in advance. Furthermore, the forecasts are rarely precise and very often wrong, which means that the forecast is also a random variable. It is true that the probability distributions of the inflows and forecasts can be determined from the historical data. The problem is not to use these data but to determine whether these historical data are still valid today given that the climate is changing.

The water released from the reservoir today will not be available tomorrow. An optimization model must therefore evaluate the tradeoffs between immediate and future use of the water, which is not always easy to do. For instance, the problem of determining when and how much electricity to sell to the spot market is difficult to solve given that the spot market price varies continuously and randomly. The reservoir management problem is not only stochastic because the inflows are stochastic, but often because the electricity demand and the spot market prices are stochastic. The reservoir management problem is probably the only optimization problem in which the input and output are stochastic and cannot be controlled in the short term.

The reservoir management problem is also difficult to solve because it is nonlinear and non-convex. It is nonlinear because the water head is a nonlinear function of the reservoir content and the generation a non-linear function of the water head and the plant discharge. The generation is in fact the product of two concave functions and is therefore not necessarily concave. The problem is also non-convex because the discharge of a generating unit can be set to zero or to a value greater than $a$ and lower than $b$, where $0 < a < b$.

The problem is even much more difficult to solve when there are several reservoirs in cascade and parallel on the river. In this case, one must also take into account the time taken by the water to travel between two neighboring reservoirs. This time can range from a few hours to several days and usually varies with the discharge of the river.

This thesis deals with the problem of finding the optimal daily operating policy of a small reservoir feeding a hydroelectric power plant and supplying water to a municipality downstream. The operating policy must respect two probabilistic constraints. The first specifies that the probability of flooding the municipality downstream must be less than a given value. The second imposes a limit to the probability of a water shortage. These probabilistic constraints are necessary because the reservoir inflow is a random variable. This variable is first assumed to be independent in the optimization problem. Next, it is assumed to be correlated to the inflows of the preceding week. The problem is also solved with inflow scenarios using the Sampling Stochastic Dynamic Programming technique.

The thesis is organized as follows. Chapter 1 contains a review of the optimization methods used for solving reservoir problems. Chapter 2 describes the problem solved in this thesis. Chapter 3 deals with stochastic processes and the synthetic generation of inflows scenarios. Finally, chapter 4 presents and discusses the numerical results obtained.

# CHAPTER 1: AN INTRODUCTION TO OPTIMIZATION TECHNIQUES IN RESERVOIR MANAGEMENT PROBLEM AND LITERATURE REVIEW

## 1.1 Reservoir System Analysis

Systems analysis models are generally classified as descriptive or prescriptive models (Simonovic, 1992). Descriptive models define what will happen if specified decisions are made. Prescriptive models determine what decisions should be made to achieve specified objectives. Simulation models are descriptive. Optimization technique such as Linear Programming, Dynamic Programming and Nonlinear Programming are prescriptive.

## 1.2 Optimization Models

Several optimization techniques have been used for solving reservoir management problems. The choice of the technique depends of the size of the optimization problem and on whether the problem is linear or nonlinear, deterministic or stochastic, convex or non-convex, separable or not separable.

Mathematical models of optimization problems usually contain one or several objective functions, constraints, and bounds on the variables. The objective can be to maximize the production of hydroelectric energy, minimize the risks of a flood, minimize the risk of a water shortage, maintain the stream flow above a certain level, etc. Constraints usually include physical characteristics of the reservoir system, minimum requirements for various purpose, and mass balances. There are usually bounds on reservoir discharges and levels.

The most popular optimization techniques are Linear Programming, Nonlinear Programming, Dynamic Programming, Optimal control and the Genetic algorithms. This literature review focuses on the application of Dynamic, Linear and Nonlinear Programming to reservoir management problems.

### 1.2.1 Linear Programming (LP)

Linear Programming is the most used technique for solving reservoir management problems. This method can be only used if the objective function and the constraints are linear. When they are not, the problem must be linearized before using LP. Piecewise linearization, first-order Taylor series expansion and iterative schemes can be used to linearize the problem (Yeh1985). There are many advantages in using LP for solving reservoir management problems, as Labadie (2004) pointed out. These include the ability of solving very large problems in a reasonable time and with little effort since commercial computer codes are available. The method also finds the global optimal solution.

Dorfman (1962) was among the first to use LP to solve a water resources management problem. Later, Windsor (1973) solved a very interesting multi-reservoir flood control problem with LP. Palmer and Holmes (1988) incorporated an LP model in the Seattle Water Department decision support system to determine optimal policies and system yield. Hiew et al. (1989) used LP for managing the eight-reservoir Colorado-Big Thompson project in Northern Colorado. Trezos (1991) showed how to combine LP, piecewise linearization and mixed-integer programming to solve a nonlinear, non-convex optimization problem. Mixed-integer linear programming was also used by Needham et al. (2000) for solving flood control problems in the Iowa and Des Moines Rivers. Separable programming was applied by Crawley and Dandy (1993) to a multi-reservoir water supply system in Australia.

### 1.2.2 Nonlinear Programming (NLP)

Linearizing a big nonlinear optimization problem to solve the problem with LP may not be the best approach when the problem is complex and has a large number of nonlinear functions, which is often the case for reservoir management problems. A better solution will likely be obtained if the problem is solved with Nonlinear Programming.

The most powerful and robust NLP methods, according to Labadie (2004), are Sequential Linear Programming (SLP), Sequential Quadratic Programming (SQP) (or Projected Lagrange method), Augmented Lagrange method (or method of multiplier) and the Generalized Reduced Gradient method (GRG). These methods require that the functions in the optimization problem be all differentiable, which is not always the case.

With SLP, the nonlinear functions are linearized around a nominal solution, using the Taylor series expansion. The linearized problem is solved with LP. If the optimal solution is different from the nominal solution, the nominal solution is set equal to the optimal solution and the functions are linearized again. The procedure is repeated until the differences between the optimal and nominal solutions become negligible. Martin (1983) has applied SLP to the problem of finding the optimal operating policy of the Arkansas-White-Red river system in Texas. Hiew (1987) compared the results obtained with SLP, GRP and SQP for this river system and concluded that SLP gives the best results. Grygier and Stedinger (1985) have also compared these three methods and came to the same conclusion. Barros et al (2003), who applied SLP to the Brazilian hydropower system, also concluded that SLP is a very efficient method for solving large nonlinear reservoir management problems.

A SQP method was applied by Tejeda-Guibert et al. (1995) to a five-reservoir system in California. Barros et al. (2003) have applied SQP to the large Brazilian hydropower system and compared the results with those obtained with SLP. To avoid the exponential growth of computer time with the number of time steps, Arnold et al. (1994) proposed a method that takes into account the special structure of the optimization problem when solving the problem with SQP.

Arnold et al. (1994) have applied SQP and the method of multipliers (MOM) to the four-reservoir Zambezi River system in southern Africa. They found out that MOM converged more rapidly than SQP.

The generalized reduced gradients (GRG) method is essentially a constrained gradient search technique that solves a reduced optimization problem with respect to the independent decision variables (Labadie, 2004). This method was used by Unver and Mays (1990) for solving the flood control problem in the Highland Lakes system in Texas. Peng and Buras (2000) also applied GRG to the five major upstream lakes in the West Branch Penobscot River.

### 1.2.3 Dynamic Programming (DP)

Dynamic Programming has been and continues to be extensively used for solving reservoir management problems. DP, which was developed by Bellman (1957), is not an optimization method but a decomposition method. It decomposes a multi-stage optimization problem into a series of one-stage problems. The popularity and success of this method is due to this fact that it gives a feedback solution and solves stochastic optimization problems.

#### 1.2.3.1 Deterministic DP and implicit stochastic optimization

Deterministic DP is used for solving reservoir management problems only when the reservoir inflows can be assumed to be known for all the period studied. One of the first applications of deterministic DP to reservoir management is that of Young (1967) who used the method to determine the optimal operating policy of a single reservoir. Hall et al. (1968) solved a similar problem with deterministic DP. Their objective was to maximize the benefit from the sale of water and energy. Allen et al. (1986) and Bhaskar and Whitlach (1980) also used deterministic DP in their papers on reservoir management. Efthymoglou (1987) solved a monthly reservoir management problem with DP in which the objective is to minimize the cost of running the thermal plants. Since the reservoir inflow cannot be forecasted long in advance, only the solution of the first month was applied. Gilles and Wunderlich (1981) succeeded to determine the optimal weekly releases of a system of 19 reservoirs with DP. Martin (1987) developed a DP algorithm capable of determining the optimal capacity of a water supply system.

Karamouz and Houck (1982) used deterministic DP, regression analysis and simulation to obtain a feedback solution to a deterministic multi-reservoir operation problem. They named their method DPR for Dynamic Programming with Regression. The paper correctly mentions that there is no guarantee that the method will give the optimal feedback solution. A combination of LP and DP was employed by Becker and Yeh (1974) for the California central Valley project. The model uses DP for selecting the optimal policy path and LP for optimizing between stages.

Unfortunately, Dynamic Programming cannot solve large problems because computational and memory requirements grow exponentially with the number of state variables. Bellman called that the «curse of dimensionality». Because of it, DP cannot generally solve optimization problems with more than four state variables in a reasonable time.

Various modifications have been performed on original DP formulation to help overcome the "curse of dimensionality". Labadie (2004) listed them as follows: Coarse Grid or Interpolation techniques, Dynamic Programming Successive Approximations (DPSA), Incremental Dynamic Programming (IDP) and Discrete Differential Dynamic Programming (DDDP).

With coarse grids, the computational and memory requirements diminish because the Bellman's functional equation is solved for fewer values of the state variables. As a result, problems of greater dimension can be solved. Naturally, the interpolation of the function over the coarser grid structure must be adequate in order to obtain a good solution. Bellman (1957) originally suggested the interpolation techniques. Later

Johnson et al. (1993) extended it to sophisticated interpolation methods using high order polynomial functions. Labadie (2004) mentioned that these methods improve the dimensionality problem, but fail to conquer it completely.

The Dynamic Programming Successive Approximations (DPSA) technique was first suggested by Bellman and Dreyfus (1962), and later generalized by Larson (1968). With this approach, multidimensional problems are decomposed into a series of one-dimensional sub-problems in such a manner that the sequence of optimization over the sub-problems converges to the solution of the original problem (Bellman, 1962). Korsak and Larson (1970) mentioned that convergence to a local optimum is guaranted when the problem is convex. There is not assurance however that the global optimal solution will be found. Yeh and Trott (1972) and Giles and Wunderlich (1981) applied this technique and its extensions to many multiple reservoir systems. Shim et al. (2002) used DPSA for solving the flood control problem of the Han River basin, Korea. Yi et al. (2003) have also applied DPSA to the lower Colorado River reservoir system. Finally, a DPSA method that adjusts the state variables in overlapping pairs was proposed by Collins (1977).

Larson (1968) developed the Incremental Dynamic Programming (IDP) technique, and Hall et al. (1969) applied it to a portion of the Central Valley Project in Northern California. The objective of the problem was to maximize the firm energy produced by the hydro plants. There are two reservoirs in this system and hence two states and decision variables in the problem. Two procedures for choosing the increments were followed. The first used the same small increment in each iteration. The second procedure reduced the size of the increments in each iteration. The second procedure was found to be more efficient.

Heidari et al. (1971) developed the technique called Discrete Differential Dynamic Programming (DDDP), which is very similar to IDP. DDDP is an iterative technique that starts with an initial trajectory and then uses DP to find a better trajectory in the neighborhood of the initial trajectory. The better trajectory becomes the initial trajectory of the next iteration. The procedure is repeated as long as the solution improves. This method was applied by Karamouz et al. (1992) to the reservoirs of the Gunpowder River Basin in Baltimore. Nopmongcol and Askew (1976) used both DPSA and DDDP to solve their reservoir management problem. They first used DPSA to find an initial solution and then used DDDP to determine the optimal solution. This method is included in the generalized dynamic programming software package CSUDP (Labadie, 2004). Note that IDP and DDDP can only be applied to deterministic problems and that there is no guarantee that the global optimal solution will be found.

Young (1976) introduced the Implicit Stochastic Optimization (ISO) technique, which consists in solving the optimization problem with DP for a large number of inflow scenarios. These scenarios may correspond to the historical data or be synthetically generated with an autoregressive model. The problem afterward is to correctly use the results obtained with DP to determine a valid closed-loop operating policy, and more specifically an operating policy that is a function of the reservoir contents and inflows. This can be done with multiple regression analysis but, as Labadie (2004) mentioned, there is no guarantee that the results obtained will be good. Roefs and Bodin (1970) used this approach to determine operational rules for a three-reservoir system. Bhaskar et al. (1980) determined linear and nonlinear release rules for a single reservoir using ISO. Performance of these release rules was verified and compared through simulation. It was found that the simple linear release rule was better than the nonlinear one.

Willis et al. (1984) determined a probabilistic operating policy for a single reservoir using Monte Carlo optimization. Karamouz (1992) did the same for a multiple reservoir system. Lund and Ferreira (1996) also applied this method to the main stem Missouri River reservoir system.

## 1.3 Explicit Stochastic Optimization

Explicit Stochastic Optimization means optimization in which the random variables, like the reservoir inflows, are explicitly represented by random variables whose probability distributions or conditional probability distributions are known.

### 1.3.1 Stochastic Linear Programming

One of the first applications of stochastic LP to reservoir management problems was that of Manne (1962). He formulated the problem of determining the optimal operating policy of a single reservoir as a Markov decision process and solved it with LP. The random variable in the Markov process was the inflow, which was discretized. The transition probabilities of the discrete Markov process in period $t$ represent the probability that the inflow be equal to value $i$ in period $t$ given that it was equal to value $j$ in period $t-1$. $i$ and $j$ represents possible values of the inflows in periods $t$ and $t-1$.

Thomas and Watermeyer (1962) extended Manne's process by defining the initial state of the system as inflow and storage rather than storage alone. Loucks (1968) solved a very interesting problem with stochastic LP, which consists in determining the optimal operating policy of a reservoir whose inflows are represented by a lag-one Markov process. The objective function of the problem is to minimize a loss function that penalizes the deviation of the storage and release from targets. The model is formulated in such a way that the decision variables are the joint probabilities of given starting reservoir volumes, inflows, and releases. The paper of Loucks et al. (1981) discussed the feasibility of applying stochastic LP to multi-reservoir systems. Jacobs et al. (1995) used stochastic LP and Bender's decomposition to determine the optimal operating policy of the hydroelectric installations of PG&E located in Northern California.

Stochastic Programming with Recourse can often be used to solve Stochastic Linear Programming problems. Dupacova (1980) applied it to water management problems. Fleten and Wallace (1998) used it to solve simultaneously the problems of determining the optimal operating policy of an aggregated reservoir and managing the risks. Seifi and Hipel (2001) solved a two-stage stochastic linear programming with recourse with an interior point method.

### 1.3.2 Chance-Constrained Programming

Chance-Constrained Programming refers to optimization problems that have probabilistic constraints, and more precisely constraints that must be satisfied with given probabilities. These constraints can easily be converted into deterministic constraints when the probability distributions of the random variables are known. When the chance-constraint problem is linear and deterministic, it can be solved with chance-constrained linear programming.

Charnes et al. (1958) originally introduced chance constraints into LP problems, and Revelle et al. (1969) were the first to use them in reservoir management problems. Datta and Houck (1984), Bhaskar and Whitlatch (1987), Dupacova et al. (1991) and Prekopa (1995) used Chance-Constrained Programming to solve multi-reservoir management problems with chance constraints.

### 1.3.3 Stochastic Dynamic Programming (SDP)

SDP is Dynamic Programming applied to stochastic optimization problems. Little (1955) used it to maximize the generation of a hydroelectric powerplant on the Columbia River. The state variables of the problem were the reservoir level and the reservoir inflow in the preceding period, which means that the inflow was represented by a lag-one Markov process. Gesford et al. (1958) solved a similar problem, but without taking the serial correlation of the inflows into account. Russel (1972) extended Gesford's work to multi-purpose reservoir operation. Loucks et al. (1981) solved SDP problems and highly recommended it for solving reservoir management problems. Several other researchers have successfully applied SDP to reservoir management problems. Stedinger et al. (1984) have provided a list of those researchers.

Hydrologic state variables are often used in SDP problems to represent the available information on the inflows. This information can be the current period's inflow, the previous period's inflow, the inflow of the past $n$ periods, etc. Many researchers, such as Esmaeil-Beik and Yu (1984), have compared the results obtained with these different hydrologic variables. Bras et al. (1983) studied the High Aswan Dam basin system and showed that incorporation of current hydrologic forecast information in an SDP model can lead to operation that is more efficient. Stedinger et al. (1984) developed a stochastic dynamic programming model that employs the best forecast of the current period's inflow to define a reservoir release policy. Tejeda-Giubert et al. (1995) compared the performance of SDP models with different state variables for three different objectives to examine the value of hydrologic information in SDP models. They mentioned that there is little difference between SDP models that employed different hydrologic state variables with an objective function stressing only energy maximization.

Karamouz and Vasiliadis (1992) used Bayesian decision theory to determine the conditional probabilities of the reservoir inflows. There SDP problem has three state variables: the reservoir storage at the beginning of the period, the reservoir inflow in the previous period, and the inflow forecast for the next period. Kim and Palmer (1997) also used a similar method to investigate the value of seasonal flow forecasts in hydropower generation. They replaced one period ahead inflow forecasts with seasonal flow forecasts using a snowmelt runoff-forecasting model as a state variable. The resulting SDP problem determines monthly operating policies for the Skagit Hydropower System, which supplies energy to the Seattle metropolitan area. Comparison of the results obtained with the Bayesian approach with those obtained with the classical approach shows that the Bayesian approach gives better results.

Labadie (2004) stated that the extension of SDP to multi-reservoir problems is annoyed by the "curse of dimensionality". The problem is even worst when the serial correlation of the inflows must be taken into account because additional state variables are needed to represent the past inflows. There are consequently very few papers on the application of SDP to multi-reservoir problems. One of those is the paper of Tejeda-Guiber et al. (1995) on the Trinity–Shasta Reservoir system of California. They investigated the value of incorporating hydrologic information into an SPD model of a two-reservoir system. The results show that the choice of the hydrologic state variable has little importance when the objective is to maximize energy. It has greater importance when the objective function is to penalize deviations from target releases and storages.

Kelman et al. (1990) developed a technique called Sampling Stochastic Dynamic Programming (SSDP) in which, unlike SDP, the stochastic structure of stream flows process is not explicitly considered. In this method, the features of the process are implicitly captured with a large number of stream flow sequences that are realizations of the annual stream flow process. The stream flow sequences observed or stochastically generated are called stream flow scenarios. However, this method mitigates, but does not eliminate the dimensionality problems of SDP and has not been applied to multi-reservoir systems.

Another approach developed to improve the dimensionality problems of SDP is to aggregate all the reservoir in the system into a single representative reservoir. TVA follows this approach with their Economy Guide Development Program by using the amount of energy in storage as the system state (Shane and Gilbert, 1982). In order to relate energy in storage to individual reservoir storage levels, TVA assumes that the optimal distribution of energy in storage maximizes the capacity of the entire system.

The National Weather Service (NWS) produces ensemble streamflow prediction (ESP) forecasts. These forecasts were used by Faber et al. (2001) in their SSDP model to optimize reservoir operations. SSDP gives better results when solved with updated ESP forecasts than with time series coupled with snowmelt-season volume forecasts.

Stedinger et al. (1984) suggested reducing the number of state variables in multi-reservoir multi-stage optimization problems by aggregating variables. The problem with state aggregation is that important information may be lost during the aggregation process, and as a result, the solution obtained may not be feasible. Stedinger (1997) proposed many other ways of improving the solution to reservoir management problems. Cervellera.C et al. (2005) determined the optimal solution of a system of 30 reservoirs with SDP. They did it by using neural network to approximate the value functions in SDP. Tingsanchali .T et al (2006) developed an optimization technique called SDPR that involves the use of DP, SDP and simulation. The technique was applied to the Kok-Ing-Nan trans-basin diversion system in Thailand. Due to the large dimension of the problem, the multi-reservoir system was decomposed into three sequentially linked reservoir subsystems.

# CHAPTER 2: FORMULATION OF THE RESERVOIR MANAGEMENT PROBLEM

The purpose of this chapter is to describe the reservoir management problem solved in this thesis, present the mathematical model of the optimization problem, and show how the problem can be solved with Dynamic Programming.

## 2.1 The reservoir management problem

The problem consists in determining the optimal operating policy of a reservoir that feeds a hydroelectric powerplant and supplies water to a municipality, as shown in Figure 2.1 below.

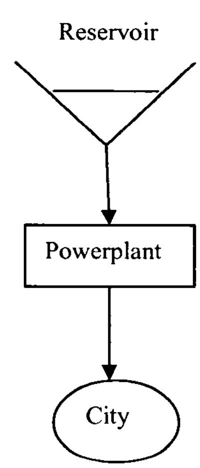


**Figure 2-1:The one-reservoir model**

The objectives are:

- To maintain the reservoir discharge equal to or greater than 40 $m^3/s$. The powerplant cannot generate electricity when the discharge is smaller than this discharge.
- To maintain the reservoir discharge equal to or less than 80 $m^3/s$, which is the maximum capacity of the powerplant.
- To minimize the risk that the municipality downstream be flooded, which occurs when the discharge exceeds 380 $m^3/s$.
- These three objectives can be satisfied by maximizing the benefit function presented in Figure 2.2. The first segment of the piecewise linear function shows that the profit increases rapidly when the discharge is increased from 0 to 40 $m^3/s$. From 40 to 80 $m^3/s$ the profit continues to increase, but at a smaller rate. Over 80 $m^3/s$ the profit decreases because the generation no longer increases and the water is wasted. Over 380 $m^3/s$ the profit decreases very rapidly because flooding occurs. It is clear that the best operating policy is to keep the reservoir discharge between 40 and 80 $m^3/s$. By changing the slopes of the segments, the operating policy will change. The idea is to change the slopes until the operating policy correctly satisfies the objectives.

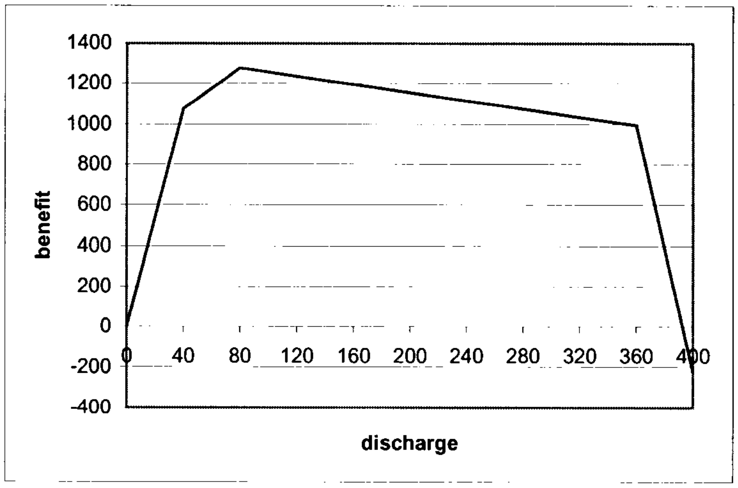


**Figure 2-2: Benefit function** $b(R_t)$

Now, if we let:

$S_t$ = the reservoir content in $hm^3$ at the beginning of day $t$, $t = 1, 2, \ldots, T$.

$Q_t$ = the reservoir inflow on $m^3/s$ in day $t$.

$R_t$ = the reservoir discharge on $m^3/s$ in day $t$.

The problem of determining the best optimal operating policy of the reservoir can be written mathematically as follows:

$$\text{Maximize } \sum_{t=1}^{T} b(R_t) + \phi(S_{T+1}) \tag{2.1}$$

Subject to the following constraints:

$$S_{t+1} = S_t + (Q_t - R_t) \cdot c \quad ; \quad t = 1, 2, \ldots, T \qquad S_1 \text{ known} \tag{2.2}$$

$$S_{t+1}^{\min} \le S_{t+1} \le S_{t+1}^{\max} \quad ; \quad t = 1, 2, \ldots, T \tag{2.3}$$

$$R_t \geq 0 \qquad ; \quad t = 1, 2, ..., T \qquad (2.4)$$

where $c$ is the factor which converts $\left(m^3/s\right) \times day$ into $hm^3$, $b(R_t)$ is the benefit for the releasing of $R_t$ $m^3/s$ from the reservoir on day $t$, and $\phi\left(S_{T+1}\right)$ is the value of the water stored in the reservoir at the end of the time horizon.

State equation (2.2) states that the reservoir content at the beginning of day $t+1$ is equal to the content at the beginning of day $t$ plus the inflow $Q_t$ minus the outflow $R_t$. Constraint (2.3) says that the reservoir content at the beginning of day $t+1$ should be equal to or greater than the lower bound $S_{t+1}^{\min}$ and equal to or smaller than the upper bound $S_{t+1}^{\max}$.

## 2.2 Dynamic Programming

Problem (2.1-2.4) can easily be solved with Dynamic Programming (DP) because there is only one reservoir on the river, which means that there is only one state variable in the problem. This state variable is $S_t$, the reservoir content at the beginning of day $t$. The control or decision variable is $R_t$. The value of this variable should be adjusted to satisfy objective (2.1) subject to constraints (2.2-2.4).

With DP, sequential or multi-stage decision problems, like problem (2.1-2.4), are converted into a series of one-stage problems. This means that a problem with

$T$ decision variables can be transformed into $T$ sub-problems, each containing only one decision variable. The values of the state variables are supposed to be known at the beginning of a stage and, hence, when the decision is taken for that stage. The problem is usually solved for a finite number of values of the state variables, meaning that the state variables are discretized when they are continuous. For reservoir management problems, the state variables are the reservoir's contents and, sometimes, a hydrologic variable containing information on past inflows.

### 2.2.1 Deterministic Dynamic Programming

DP can solve deterministic and stochastic optimization problems. For the reservoir management problem, for instance, the problem is deterministic when the reservoir inflows are assumed to be known. It is stochastic when the reservoir inflows are represented by stochastic processes.

When DP is applied to the daily reservoir operation, the problem is broken down into $T$ problems, where $T = 365$, as shown below.

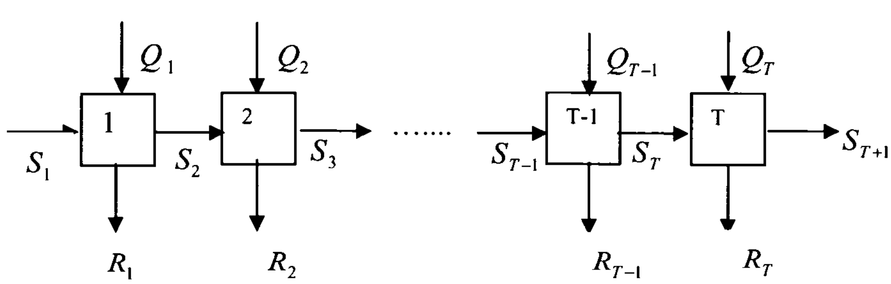


**Figure 2-3: Stages in reservoir operation**

The problem consists in solving the following functional equation:

$$f_1(S_1) = \max_{R_1,\ldots,R_T} \sum_{t=1}^{T} b(R_t) \tag{2.5}$$

$$= \max_{R_1}\left[ b(R_1) + \max_{R_2,\ldots,R_T} \sum_{t=2}^{T} b(R_t) \right] = \max_{R_1}\left[ b(R_1) + f_2(S_2) \right] \tag{2.6}$$

$$f_2(S_2) = \max_{R_2}[b(R_2) + f_3(S_3)] \tag{2.7}$$

.

$$f_T(S_T) = \max_{R_T}[b(R_T) + f_{T+1}(S_{T+1})] \tag{2.8}$$

$$f_{T+1}(S_{T+1}) = \phi(S_{T+1}) \tag{2.9}$$

In brief, the problem consists in solving functional equation (2.10) backward in time, starting in period $T$ with $f_{T+1}(S_{T+1})$ defined in (2.9).

$$f_t(S_t) = \max_{R_t}[b(R_t) + f_{t+1}(S_{t+1})] \tag{2.10}$$

In stage $t$, a release $R_t$ is selected to maximize the sum of the current benefit, $b(R_t)$, and the future benefit, $f_{t+1}(S_{t+1})$.

### 2.2.2 Stochastic Dynamic Programming

Stochastic Dynamic Programming (SDP) is an extension of DP to stochastic optimization problems. For the reservoir management problem, SDP is used when the reservoir inflow is supposed to be a random variable with a known probability distribution. The problem becomes then to maximize the expected value of the benefits and not the benefits themselves.

Mathematically, the problem consists in solving recursive equation (2.11) backward in time, starting in period $T$ with $f_{T+1}(S_{T+1})$ defined in (2.9).

$$f_t(S_t) = \max_{R_t} \underset{Q_t}{E}\left\{b(R_t) + f_{t+1}(S_{t+1})\right\} \tag{2.11}$$

subject to constraints (2.2-2.4). The symbol $\underset{Q_t}{E}$ represents the expected value with respect to variable $Q_t$. Since $\underset{Q_t}{E}$ is after the "max" in (2.11), the value of $R_t$ will be chosen before $Q_t$ becomes known. Let $R_t^{opt}$ be this value. Depending on the inflow, the inequalities in (2.3) may not be satisfied with this solution. This will happen if:

$$S_t + \left(Q_t - R_t^{opt}\right) \cdot c > S_{t+1}^{\max} \tag{2.12}$$

In this case, the discharge $R_t$ must be set to:

$$R_t = Q_t + \left(S_t - S_{t+1}^{\max}\right)/c \tag{2.13}$$

In equalities (2.3) will also not be satisfied if:

$$S_t + \left(Q_t - R_t^{opt}\right) \cdot c < S_{t+1}^{\min} \tag{2.14}$$

and, in this case, $R_t$ will have to be set to:

$$R_t = Q_t + \left(S_t - S_{t+1}^{\min}\right)/c \tag{2.15}$$

The problem with the last equation is that $R_t$ may be negative, which of course is impossible. In other word, if $Q_t + \left(S_t - S_{t+1}^{\min}\right)/c < 0$, there is no solution to the problem. In this case, there is no choice but to replace constraint $S_{t+1} \geq S_{t+1}^{\min}$ by a penalty function in the objective function. Let this penalty function be:

$$p_t\left(S_{t+1}\right) = h \cdot \max\left(0, S_{t+1}^{\min} - S_{t+1}\right) \tag{2.16}$$

where $h$ is a pre-selected constant, and let:

$$B_t\left(S_{t+1}, R_t\right) = b\left(R_t\right) - p_t\left(S_{t+1}\right). \tag{2.17}$$

The problem becomes then:

$$f_t(S_t) = \max_{R_t} \underset{Q_t}{E}\left\{B_t\left(S_{t+1}, R_t\right) + f_{t+1}(S_{t+1})\right\} \tag{2.18}$$

subject to constraints (2.2), (2.4) and:

$$0 \leq S_{t+1} \leq S_{t+1}^{\max} \tag{2.19}$$

In practice, the reservoir inflow is usually known when the reservoir discharge is adjusted, so that the optimization problem is the following:

$$f_t(S_t) = \underset{Q_t}{E}\left[\max_{R_t}\left\{B_t\left(S_{t+1}, R_t\right) + f_{t+1}(S_{t+1})\right\}\right] \tag{2.20}$$

Since the expected sign $\underset{Q_t}{E}$ is before the "max" in the last equation, $Q_t$ is known when $R_t$ is set. We still need the penalty function (2.16) because the lower bound $S_{t+1}^{\min}$ may still be violated. This will be the case if $S_t^{\min}$ is small, $S_{t+1}^{\min}$ is big and the inflow is smaller than $\left(S_{t+1}^{\min} - S_t^{\min}\right)/c$. The solution to problem (2.20) is of the feedback type because it is a function of $S_t$, the reservoir content at the beginning of stage $t$, and of $Q_t$, the inflow in stage $t$. This function is usually designated by $R_t^{opt}\left(S_t, Q_t\right)$.

The inflow in stage $t$ is often correlated to that of stage $t-1$. When the stage is short, like day, the inflow in stage $t$ may even be correlated to those of many previous days. The correlation between the inflows in stages $t$ and $t-1$, called the lag-one autocorrelation, can be taken into account in problems (2.18) and (2.20) by adding a new state variable representing $Q_{t-1}$. Equation (2.20) becomes:

$$f_t(S_t, Q_{t-1}) = \underset{Q_t|Q_{t-1}}{E}\left[\max_{R_t}\left\{B_t\left(S_{t+1}, R_t\right) + f_{t+1}(S_{t+1}, Q_t)\right\}\right] \tag{2.21}$$

where $\underset{Q_t|Q_{-1}}{E}$ represents the conditional expectation. When $Q_t$ is correlated not only to $Q_{t-1}$, but to $Q_{t-2}, Q_{t-3}, \ldots, Q_{t-n}$, the following recursive equation must be solved:

$$f_t(S_t, Q_{t-1}, \ldots, Q_{t-n-1}) = \underset{Q_t|Q_{t-1}, \ldots, Q_{t-n-1}}{E}\left[\max_{R_t}\left\{B_t\left(S_{t+1}, R_t\right) + f_{t+1}(S_{t+1}, Q_t, \ldots, Q_{t-n})\right\}\right] \tag{2.22}$$

Since the computer time and memory space increase exponentially with the number of state variables, equation (2.22) will not be solved in a reasonable time if $n$ is greater than 3.

Not only the past inflows but the current inflow and an inflow forecast can be used as a hydrologic state variable in the DP recursive equation. The current inflow and the inflow forecast have in fact been used by many authors (Stedinger et al., 1984; Kelman et al., 1990; Maceira and Kelman, 1991; Karamouz and Vasiliadis, 1992; Tejeda-Guibert et al., 1995; Kim and Palmer, 1997).

### 2.2.3 Sampling Stochastic Dynamic Programming

Kelman et al. (1990) proposed a variation of SDP called Sampling Stochastic Dynamic Programming (SSDP) which employs a set of inflow scenarios to represent future inflows. Faber and Stedinger (2001) solved the following problem with this method:

For each scenario $i$ $(i = 1,2,...,I)$, solve the following two problems in stage $t$:

$$[1] \quad \underset{R_t}{Maximize} \ \{B_t(S_{t+1}, R_t) + \sum_{j=1}^{I} f_{t+1}(S_{t+1}, j).\Pr(j|\, i)\}$$

subject to constraints (2.2), (2.4) and (2.19). Let $R_t^i$ be the solution.

$$[2] \quad \text{Set} \quad f_t\left(S_t, i\right) = B_t\left(S_{t+1}, R_t^i\right) + f_{t+1}\left(S_{t+1}, i\right)$$

where $\Pr\left(j|i\right)$ is the conditional probability of scenario $j$ given scenario $i$.

In this SSDP formulation, the hydrologic state variable is $i$, the number of the inflow scenario. A simple choice for $\Pr(j|i)$ is $\Pr(j|i)=1$ for $j=i$ and $\Pr(j|i)=0$ otherwise. This choice is equivalent to performing a deterministic optimization with scenario $i$. Another simple choice is to assume that $\Pr(j|i)$ is the same for all $j$ and $i$, that is $\Pr(j|i)=1.0/I$. In this thesis, the conditional probability $\Pr(j|i)$ was determined using the set of historical inflow scenarios.

# CHAPTER 3: AN INTRODUCTION TO STOCHASTIC PROCESSES AND THEIR APPLICATION TO RESERVOIR MANAGEMENT PROBLEMS

## 3.1 Introduction

It is very difficult to correctly forecast the weather, and hence the precipitation, a long time in advance. As a result, the reservoir inflows resulting from the precipitations are usually not known several days in advance. This does not mean however that there exists no information on the inflow that might occur in six months from now. If we assume that the characteristics of future inflows will resemble those of the past, the historical inflow data can be used to determine the probability distribution of future inflows.This chapter deals with the determination of the inflow probability distributions using stochastic processes and time series analysis.

## 3.2 Stochastic Processes

Let $X$ denote a variable. If the value of this variable is known, the variable is said to be deterministic. If the value is not known, $X$ is said to be a random variable provided that the probability distribution of this variable is known or can be determined from the historical data.

Suppose that $X$ is a random variable that changes with time and let $X = X_t$ at time $t$. The series $\{X_1, X_2, ..., X_T\}$ is called a discrete stochastic process. A continuous stochastic process varies continuously with time and is usually represented by $\{X_t \; ; \; 0 \le t \le T\}$. This thesis uses discrete stochastic processes only.

### 3.2.1 Definitions

This section defines the statistics used in this document. The symbols $\mu_t = E(X_t)$ and $\sigma_t^2 = Var(X_t)$ represent the expected value and variance of random variable $X_t$. The symbol:

$$\gamma_{t,k} = \text{cov}(X_t, X_{t+k}) = E[(X_t - \mu)(X_{t+k} - \mu)] \tag{3.1}$$

represents the covariance between $X_{t+k}$ and $X_t$. A normalized quantity that is more convenient to deal with than $\gamma_{t,k}$ is the autocorrelation coefficient:

$$\rho_{t,k} = \frac{\gamma_{t,k}}{\gamma_{t,0}} \tag{3.2}$$

where $-1 \le \rho_{t,k} \le 1$ . When $\rho_{t,k} = 0$, there is no linear relation between $X_{t+k}$ and $X_t$. When $\rho_{t,k} = 1$, the variable $X_{t+k}$ is a deterministic function of $X_t$. When $-1 \le \rho_{t,k} \le 1$, $X_{t+k}$ is said to be correlated to $X_t$.

A stochastic process is said to be stationary when the statistics do not change with time. In this case, $E(X_t) = \mu$, $Var(X_t) = \sigma^2$ and $\text{cov}(X_t, X_{t+k}) = \gamma_k$ for $t = 1,2,...,T$.

### 3.2.2 Sample statistical characteristics of time series

The statistics determined from historical data cannot be claimed to be the true statistics because the obtained statistics would likely be different would the historical data be longer or shorter. Historical data, or data samples, determine sample statistics and not the real statistics of the random variables.

The mean $\overline{X}_t$ of a sample of $N$ data for period $t$ is calculated with:

$$\overline{X}_t = \frac{1}{N}\sum_{n=1}^{N} X_{t,n} \tag{3.3}$$

The sample mean $\overline{X}_t$ is an estimator of the population mean $\mu_t$ for period $t$. The sample variance $S_t^2$ is determined with:

$$S_t^2 = \frac{1}{N-1}\sum_{n=1}^{N}(X_{t,n} - \overline{X}_t)^2 \tag{3.4}$$

The square root of $S_t^2$ is called the standard deviation. The standard deviation measures the spread of the data around the mean $\overline{X}_t$. A small $S_t$ means that the values $X_{t,1}, X_{t,2}, ...., X_{t,N}$ do not deviate much from $\overline{X}_t$, while a large $S_t$ generally means that values $X_{t,1}, X_{t,2}, ...., X_{t,N}$ have a large spread around $\overline{X}_t$.

The sample skewness coefficient is equal to:

$$g_t = \frac{N\sum_{n=1}^{N}(X_{t,n} - \overline{X}_t)^3}{(N-1)(N-2)\ S_t^3} \tag{3.5}$$

This coefficient measures the asymmetry of the probability distribution. When $g_t = 0$ the probability distribution is symmetric about $\overline{X}_t$. When $g_t < 0$ the distribution is skewed to the right and when $g_t > 0$ it is skewed to the left.

The autocorrelation coefficient is estimated by:

$$r_{t,k} = \frac{c_{t,k}}{c_{t,0}} \tag{3.6}$$

with $c_{t,k}$ equal to:

$$c_{t,k} = \frac{1}{N}\sum_{n=1}^{N-k}(X_{t,n} - \overline{X}_t)(X_{t+k,n} - \overline{X}_{t+k}), \qquad 0 \le k \le N \tag{3.7}$$

The coefficient $r_{t,k}$ is an estimator of the population autocorrelation coefficient $\rho_{t,k}$. The curve showing $r_{t,k}$ versus $k$ is called a correlogram. It is useful for determining the number of lags in an autoregressive model of the time series.

## 3.3 Identification of Distribution and Normalization

It is not sufficient to determine the statistics of the time series, like the mean, variance, skewness, and autocorrelation. One must also determine the probability distribution of the time series. The most commonly used distributions in hydrology are the normal, lognormal and gamma distributions.

### 3.3.1 Normal Distribution

The normal distribution is the most extensively used distribution in probability and statistics.

Suppose that the random variable $X$ is normally distributed. The probability density function of $X$ is then equal to:

$$f(X) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\{-\frac{1}{2}(\frac{X-\mu}{\sigma})^2\} \qquad -\infty < X < +\infty \qquad (3.8)$$

where $\mu$ represents the mean and $\sigma$ the standard deviation. There may be a problem with using this distribution for reservoir inflows since these cannot be negative. If other words $X \geq 0$ if $X$ represents the reservoir inflow.

### 3.3.2 Log-normal distribution

The second distribution that is also extensively used in hydrology is the log-normal distribution. This distribution assumes that the logarithms of the data are normally distributed. The log-normal distribution is positively skewed, which is often a characteristic of the streamflows, and can only be used for $X \geq 1$.

If $X$ has a log-normal distribution with two parameters, its probability density function is:

$$f(X) = \frac{1}{\sqrt{2\pi}\, X \sigma_y} \exp\{-\frac{1}{2}[\frac{\ln(X) - \mu_y}{\sigma_y}]^2\} \tag{3.9}$$

where $\mu_y$ and $\sigma_y$ represent the mean and standard deviation of the variable $y = \ln(X)$. All properties of the normal distribution are applicable to this distribution when the data are converted to logarithmic form.

### 3.3.3 Gamma Distribution

The gamma distribution is mainly used when the skewness of the data is so pronounced that the lognormal distributions do not give adequate results.

When the random variable $X$ is assumed to have a two-parameter gamma distribution, its probability density function is given by:

$$f(X) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{X}{\beta}} \tag{3.10}$$

where $\alpha$ and $\beta$ are the parameters and $\Gamma(.)$ the gamma function operator.

### 3.3.4 Normalization

As mentioned before, it is easier to deal with the normal probability distribution than with skew distributions because the normal distribution is entirely represented by its first two moments. The normal probability distribution is almost always used in time series analysis because a sum of normally distributed random variables is normally distributed. When the random variables are not normally distributed, the best policy is to normalize the data. For instance, if the set of data $X_1, X_2, ..., X_N$ is not normally distributed, a mapping $\Gamma$ should be applied to the data to obtain a set $Y_1, Y_2, ..., Y_N$ that is normally distributed. In other words, the mapping must be such that $Y = \Gamma(X)$ is normally distributed. The function $\Gamma$ depends on the skewness of the original data. This function is selected in this thesis with the Filliben test. For instance, if the data have a lognormal distribution, the function $\Gamma$ is set equal to $\ln(\ )$. In other words, $Y = \ln(X)$.

## 3.4 Time Series Modeling

Let $X_{t,n}$ be the reservoir inflow recorded on day $t$ of year $n$, $n = 1, 2, ..., N$. Time series modeling consists in finding a mathematical model for the inflows on day $t$ using the data $X_{t,n}$. The model must be such that the synthetic inflows generated with it have the same statistics than the data $X_{t,n}$. More precisely, the synthetic inflows must have the same mean, variance and skewness coefficient than the original data. Furthermore, if the inflows on day $t$ are correlated to those of days $t-1, t-2, ..., t-p$, which is usually the case, the synthetic inflows must also be correlated to those of the $p$ preceding days. The most commonly used models for modeling the inflows are the autoregressive (AR) and autoregressive-moving-average (ARMA) models.

### 3.4.1 AR models

Let $Z_t = X_t - \mu_t$, where $X_t$ is a random variable representing the inflow on day $t$ and $\mu_t$ the mean of $X_t$, and suppose that $X_t$ is normally distributed. If $X_t$ is correlated to the inflows of the $p$ preceding days and the relations between $X_t$ and $X_{t-1}, X_{t-2}, \ldots, X_{t-p}$ are linear, then we can assume that:

$$Z_t = \phi_{t,1} Z_{t-1} + \phi_{t,2} Z_{t-2} + \ldots + \phi_{t,p} Z_{t-p} + \varepsilon_t \tag{3.11}$$

where $\phi_{t,1}, \phi_{t,2}, \ldots, \phi_{t,p}$ are autoregression coefficients and $\varepsilon_t$ an independent normally distributed random variable with mean zero and variance $\sigma_\varepsilon^2$. The auroregression coefficients can be determined by the methods of moments, least square or maximum likelihood.

### 3.4.2 ARMA models

The random variable $Z_t$ could also possibly be modeled by the following moving average equation:

$$Z_t = \varepsilon_t - \sum_{j=0}^{q} \theta_{t,j}\varepsilon_{t-j} \tag{3.12}$$

where $\theta_{t,0}, \theta_{t,1}, ..., \theta_{t,q}$ are the parameters of the model and $\varepsilon_t, \varepsilon_{t-1}, ...$ are the error terms. The error terms $\varepsilon_t$ are generally assumed to be independent identically-distributed random variables sampled from a normal distribution with zero mean and variance $\sigma^2$. Another possibility would be to use an ARMA ($p$, $q$) model like the following one

$$Z_t = \phi_{t,1}Z_{t-1} + ... + \phi_{t,p}Z_{t-p} + \varepsilon_t - \theta_{t,1}\varepsilon_{t-1} - ... - \theta_{t,q}\varepsilon_{t-q} \tag{3.13}$$

This model contains the AR($p$) and MA($q$) models,

# CHAPTER 4: CASE STUDY AND RESULTS

## 4.1 Introduction

This chapter presents the numerical results obtained for the reservoir management problem described in section 2.1. The data and characteristics of the system studied correspond to those of the Kenogami Lake in the Saguenay region. There are however some differences. Kenogami Lake feeds two rivers, Aux-Sables and Chicoutimi, and not only one river. Furthermore, there are three powerplants on these two rivers and not only one plant. The reservoir constraints are however similar to those of Kenogami Lake.

### 4.1.1 Inflows

There exists a historical record of 84 years of daily inflows for the Kenogami Lake. Figure 4-1 gives the minimum, mean and maximum recorded inflows in each day of the year .The mean annual inflow is equal to 76 $m^3/s$. Figure 4-2 shows the correlograms of the inflows for the first day of each month. It is important to notice that the lag-one correlation coefficient is always greater than 0.8, which is very high. In fact the mean of the daily lag-one correlation coefficients is equal to 0.906461.

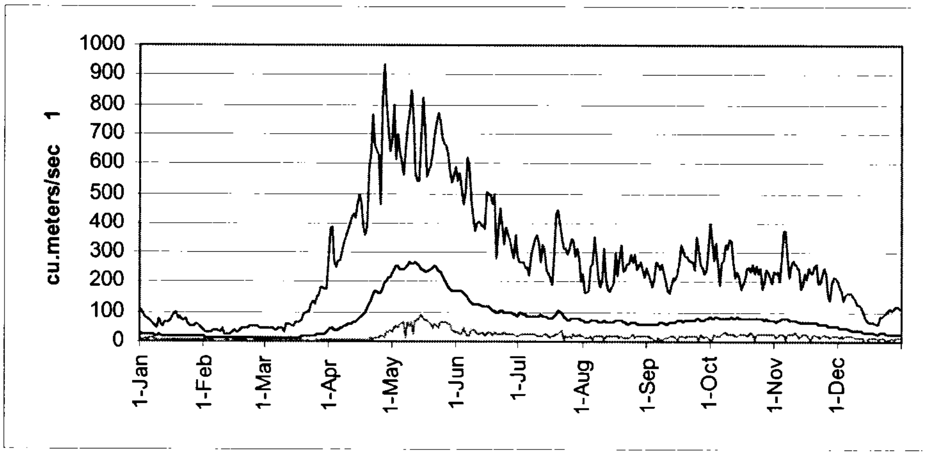


**Figure 4-1 Minimum, mean and maximum daily inflows**

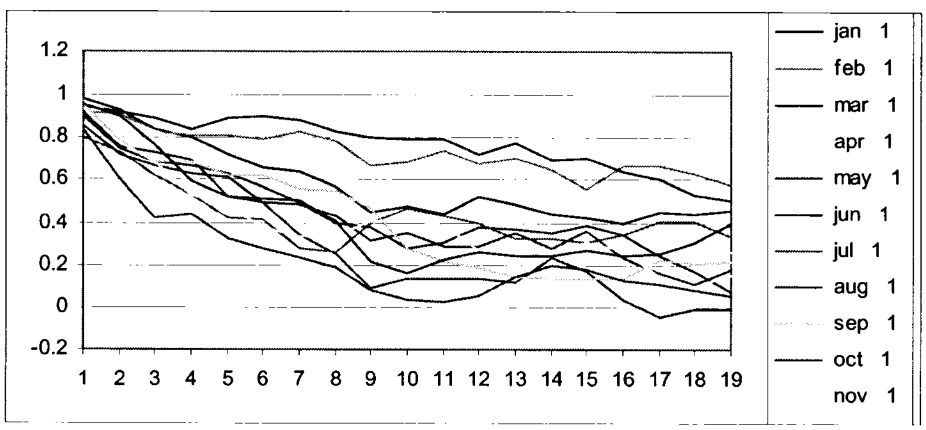


**Figure 4-2 Correlograms of the first day of each month**

Figure 4-3 shows the computed coefficients of skewness for the daily inflows. According to *Snedecor and Cochran* [1967, p. 552], the skewness coefficient should be smaller than 0.432 to assume that the data are normally distributed when the sample size is equal to 80 and the probability level is set to 0.10.

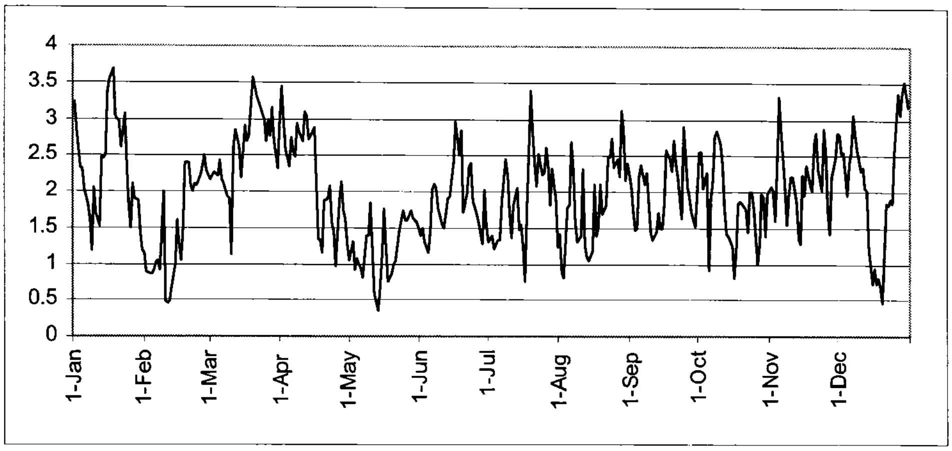


**Figure 4-3 Daily skewness coefficient**

There is consequently no day in Figure 4-3 where the inflows can be supposed to be normally distributed.

Table 4-1 presents inflows statistics for the first day of each month. Column 2 in this table gives the mean of 84 years of daily inflows for the first day of each month. Columns 3 and 4 give respectively the standard deviation and skewness coefficient of 84 years of daily inflows for the first day of each month .Finally column 5 presents the most appropriate distribution for the inflows selected by the Filliben test.

**Table 4-1 Inflows Statistics**

| The first day Of month | Mean | Standard Deviation | Skewness Coefficient | Distribution |
|---|---|---|---|---|
| JAN | 25 | 15 | 3.2240 | Log-normal-3 |
| FEB | 16 | 6 | 1.1493 | Puissance |
| MAR | 14 | 6 | 2.1686 | Log-norm-2 |
| APR | 46 | 67 | 3.4311 | Log-norm-2 |
| MAY | 227 | 130 | 1.0473 | Puissance |
| JUN | 169 | 103 | 1.3745 | Log-norm-2 |
| JUL | 95 | 55 | 1.3203 | Log-norm-2 |
| AUG | 69 | 40 | 1.4086 | Log-norm-2 |
| SEP | 60 | 41 | 2.0781 | Puissance |
| OCT | 85 | 62 | 2.0060 | Log-norm-2 |
| NOV | 68 | 34 | 2.0886 | Log-norm-2 |
| DEC | 50 | 31 | 2.8010 | Log-norm-3 |

Since the random variables in autoregressive equation $Q_t = \phi_{0,t} + \phi_{1,t} Q_{t-1}$ must be normally distributed, the inflow data were normalized using the logarithmic transformations with two or three parameters or the Gamma transformation. The choice of one of the three transformations was made by applying the Filliben test.

Table 4-2 presents normalized inflow statistics for the first day of each month. Column 2 in this table gives the mean of 84 years of normalized inflows for the first day of each month. Columns 3 and 4 give the standard deviation, and skewness coefficient of 84 years of normalized inflows for the first day of each month. Finally columns 5 and 6 present coefficient of the autoregressive equation, and the residual standard deviation of the normalized inflows.

**Table 4-2 Normalized inflows statistics**

| The first day Of month | Mean | Standard Deviation | Skewness Coefficient | Coefficients of Autoregressive Equation | Residual Standard Deviation |
|---|---|---|---|---|---|
| JAN | 2.66 | 0.64 | 0.1322 | 0.934 | 0.286 |
| FEB | 2.51 | 0.84 | 0.1675 | 0.872 | 0.129 |
| MAR | 2.54 | 0.41 | 0.2408 | 0.939 | 0.215 |
| APR | 3.30 | 0.90 | 1.0669 | 1.042 | 0.225 |
| MAY | 11.11 | 3.85 | 0.0189 | 0.838 | 0.290 |
| JUN | 4.96 | 0.59 | -0.0345 | 0.958 | 0.207 |
| JUL | 4.40 | 0.55 | 0.0297 | 1.030 | 0.329 |
| AUG | 4.09 | 0.55 | 0.0225 | 0.927 | 0.212 |
| SEP | 2.76 | 0.87 | -0.0158 | 0.861 | 0.186 |
| OCT | 4.26 | 0.60 | 0.1661 | 0.980 | 0.358 |
| NOV | 4.12 | 0.45 | -0.0149 | 0.762 | 0.162 |
| DEC | 3.50 | 0.58 | 0.6764 | 0.791 | 0.191 |

### 4.1.2 The benefit function

As mentioned in chapter 2, the benefit $b(R_t)$ of releasing $R_t$ m$^3$/s was conceived so that the optimal reservoir discharge will usually lie between 40 and 80 m$^3$/s (Figure 2-2). Below 40 m$^3$/s, the run-of-river plant downstream the reservoir cannot operate. Above 80 m$^3$/s, the water is spilled because the plant runs at its maximum capacity. The benefit function penalizes those spillages. When the discharge exceeds 380 $m^3/s$, flooding occurs, which explains why the benefit decreases rapidly and becomes negative.

## 4.2 The optimization problem

The optimal operating policy of the reservoir was first determined with Sampling Dynamic Programming, and more specifically by solving the following optimization problem in stage $t$:

$$\underset{R_r}{\text{Maximize}} \left\{ B_t\left(S_{t+1}, R_t\right) + \sum_{j=1}^{N} f_{t+1}\left(S_{t+1}, Q_{j,t}\right) \cdot \Pr\left(Q_{j,t} \middle| Q_{i,t-1}\right) \right\} \tag{4-1}$$

subject to the following constraints:

$$S_{t+1} = S_t + (Q_{j,t} - R_t) \cdot c \tag{4-2}$$

$$S_{t+1}^{\min} \leq S_{t+1} \leq S_{t+1}^{\max} \tag{4-3}$$

$$R_t \geq 0 \tag{4-4}$$

where $Q_{j,t}$ corresponds to the inflow of scenario $j$ in stage $t$. The function $\Pr\left(Q_{j,t} \middle| Q_{i,t-1}\right)$ gives the probability that the inflow be equal to $Q_{j,t}$ in stage $t$ given that it was equal to $Q_{i,t-1}$ in stage $t-1$.

As it was shown in section 2, problem (4-1)-(4-4) must be solved backward in time, and therefore for $t = 365,\ 364, \ldots, 2,\ 1$ when the problem is solved on a daily time basis. Let $R_t^{opt}\left(S_t, Q_{i,t}\right)$ be the solution to problem (4-1)-(4-4), and set:

$$f_t\left(S_t, Q_{i,t-1}\right) = B_t\left(S_{t+1}, R_t^{opt}\left(S_t, Q_{i,t}\right)\right) + f_{t+1}\left(S_{t+1}, Q_{i,t}\right) \tag{4-5}$$

Once equation (4-5) has been solved for $i = 1, 2, ..., N$, where $N$ is the number of scenarios, the procedure consists in returning to (4-1) and solves the problem for $t = t - 1$. Equations (4-1) and (4-5) were solved for 51 equidistant values of $S_t$ in the interval $\left[0,\ S_k^{\max}\right]$. The optimization problem was solved for $N$ equal to 100, 200 and 300 so as to determine if the solution improves with the number of scenarios.

Recall that:

$$B_t\left(S_{t+1}, R_t\right) = b\left(R_t\right) + \alpha \cdot \max\left(0,\ S_{t+1}^{\min} - S_{t+1}\right) \tag{4-6}$$

The role of the second term in the right-hand side of (4-6) is to keep the level of the reservoir high enough during the summer, and preferably above $S_{k+1}^{\min}$, to permit navigation and other recreational activities. The symbol $\alpha$ is a pre-selected constant. The optimal operating policy determined by (4-1) is not stored in the computer memory. Only the functions $f_t(S_t, Q_{i,t-1})$, $t = 1, ..., 365$, are stored. Those are used to simulate the operation of the reservoir over a period of $N$ years.

## 4.3 Simulation

Once the optimal operating policy of the reservoir has been determined, the next step consists in simulating the operation of the reservoir over a long period of time with the optimal operating policy. The simulations are usually done with the historical data in order to compare the results obtained with the optimal operating policy to the historical results. This was not done since there are no historical results for our problem.

Simulations are also useful to verify the mathematical model. For instance, one may wonder whether the benefit function $b(R_t)$ shown in Figure 2.2 is the best function for the problem. The only way to find out is to simulate the operation of the reservoir over many years and analyze the results, and hence determine:

- The number of floods
- The number of days that the reservoir discharge is smaller that $40\ m^3/s$
- The number of days that the reservoir level is lower than $S_t^{\min}$ in the summer
- The volume of water spilled

If the results are not acceptable, the piecewise linear function in Figure 2.2 and/or the parameter $\alpha$ in (4-6) should be modified. The procedure of simulating the operation of the reservoir and modifying the mathematical model should be repeated as long as the results are not acceptable.

It was assumed in this study that the mathematical model is valid. The simulations were done as follows. Suppose that we are at the beginning of day $t$ and that the inflow yesterday corresponds to that of scenario $i$. The discharge for the coming day is then set equal to the value of $R_t$ determined by (4-1). When the value of $S_{t+1}$ is not exactly equal to one of the 51 values of $S_{t+1}$ used to solve (4-1) or when the value of $Q_t$ is not exactly equal to one of the $N$ scenarios, linear interpolation is used to approximate the function $f_{t+1}(S_{t+1}, Q_t)$.

## 4.4 Numerical results

Table 4-3 presents results of the reservoir simulations performed with the method described above. The simulations were done over periods of 100, 200 and 300 years with different sets of inflow scenarios. Option 1 in Table 4-3 means that problem (4-1) was solved with $\Pr\left(Q_{j,t} \middle| Q_{i,t-1}\right) = 1/N$ for all $j$. In other words, it was assumed that the probability of moving from scenario $j$ to any another scenario at the end of stage $t$ is equal to $1/N$. Option 2 used the transition probabilities determined with the autoregressive equation and hence different probabilities for different transitions. $Q_{j,t}$ and $Q_{i,t-1}$ satisfy the following autoregressive equation :

$Q_{j,t} = \phi_{0,t} + \phi_{1,t} Q_{i,t-1} + \varepsilon_t$ where $\varepsilon_t$ is the error term. So in option 2, probability $\Pr(Q_{j,t} | Q_{i,t-1})$ is equal to the probability that $\varepsilon_t = Q_{j,t} - \phi_{0,t} - \phi_{1,t} Q_{i,t-1}$.

Column 3 in Table 4-3 gives the mean annual benefit for the $N$ years of simulation. Column 4 gives the mean annual penalty cost for violating the constraint on the minimum level of the reservoir. Column 5 gives the mean annual value of the water spilled. Recall that water is spilled when $R_t \geq 80$ .Column 6 gives the average number of days in a year that the reservoirs discharge is lower than 80 mcs. Finally column 7 shows average number of days in a year that the reservoir discharge exceeds 380 mcs.The differences in the benefits between Option1 and Option 2 clearly show that using the same probability for all the transitions does not give the best results.

**Table 4-3 Results for different scenario**

| Numbers of Scenarios | Transition probability | Mean annual values of | | | |
|---|---|---|---|---|---|
| | | Benefit | Penalty costs | discharge lower than 80 mcs. | discharge more than 380 mcs. |
| m=100 | Option 1 | 397092 | 19324 | 311 | 4 |
| | Option 2 | 412600 | 0 | 352 | 0 |
| m=200 | Option1 | 394546 | 17980 | 330 | 1 |
| | Option 2 | 406630 | 0 | 349 | 0 |
| m=300 | Option 1 | 388928 | 34051 | 321 | 1 |
| | Option 2 | 405362 | 0 | 351 | 0 |

In order to analyze the effects of slope of curve $b(R_t)$, the slope for two segments of the piecewise linear curve were changed (Figure 2-2).By increasing the slope of the _first_ segment of the piecewise linear curve of Figure 2-2, the benefit improved. On the other hand the decreasing for the same slope resulted in a lower benefit. By increasing the slope of the _last _ segment of the piecewise linear curve of figure 2-2, the benefit improved. Decreasing in this slope resulted in a lower benefit.

Table 4-4 presents the results of these changes in the _first_ segment of the piecewise linear curve of Figure 2-2.In order to compare results, the results of table 4-2 were also presented in section without.

**Table 4-4: Result obtained by changing the slope of the first segment of figure 2-2**

| Numbers of Scenarios | Transition probability | slope | Mean annual values of | | | |
|---|---|---|---|---|---|---|
| | | | Benefit | Penalty costs | discharge is lower than 80 mcs. | Discharge exceeds 380 mcs. |
| M=100 | Option 1 | Increase | 419562 | 19419 | 304 | 3 |
| | | decrease | 394620 | 19324 | 307 | 3 |
| | | without | 397092 | 19324 | 311 | 4 |
| | Option 2 | increase | 425280 | 0 | 351 | 1 |
| | | decrease | 399926 | 0 | 342 | 1 |
| | | without | 412600 | 0 | 352 | 0 |
| M=200 | Option 1 | increase | 417163 | 17813 | 332 | 3 |
| | | decrease | 392071 | 17896 | 330 | 3 |
| | | without | 394546 | 17980 | 330 | 1 |
| | Option 2 | increase | 419365 | 0 | 349 | 0 |
| | | decrease | 393898 | 0 | 353 | 0 |
| | | without | 406630 | 0 | 349 | 0 |
| M=300 | Option 1 | increase | 401073 | 34421 | 302 | 3 |
| | | decrease | 376012 | 34151 | 306 | 3 |
| | | without | 388928 | 34051 | 321 | 1 |
| | Option 2 | increase | 417937 | 2869 | 330 | 2 |
| | | decrease | 392690 | 2815 | 330 | 1 |
| | | without | 405362 | 0 | 351 | 0 |

Table 4-5 presents results of changes in the _last_ segment of the piecewise linear curve of Figure 2-2.

**Table 4-5: Result obtained by changing the slope of the last segment of figure 2-2**

| Numbers of Scenarios | Transition probability | slope | Mean annual values of | | | |
|---|---|---|---|---|---|---|
| | | | Benefit | Penalty costs | discharge is lower than 80 mcs. | discharge exceeds 380 mcs. |
| M=100 | Option 1 | increase | 419493 | 13368 | 317 | 3 |
| | | decrease | 394418 | 23561 | 305 | 3 |
| | | without | 397992 | 19324 | 311 | 4 |
| | Option 2 | Increase | 421395 | 0 | 353 | 0 |
| | | Decrease | 403545 | 0 | 349 | 1 |
| | | without | 412600 | 0 | 352 | 0 |
| M=200 | Option 1 | Increase | 416970 | 12123 | 333 | 3 |
| | | Decrease | 391007 | 19939 | 328 | 4 |
| | | Without | 394546 | 17980 | 330 | 1 |
| | Option 2 | Increase | 415445 | 0 | 349 | 0 |
| | | decrease | 398586 | 0 | 351 | 1 |
| | | Without | 406630 | 0 | 349 | 0 |
| M=300 | Option 1 | Increase | 401073 | 34421 | 318 | 3 |
| | | Decrease | 376012 | 34151 | 319 | 3 |
| | | without | 388928 | 34051 | 321 | 1 |
| | Option 2 | increase | 415369 | 2339 | 328 | 10 |
| | | Decrease | 396560 | 3571 | 330 | 10 |
| | | without | 405362 | 0 | 351 | 0 |

The same optimization problem was solved with Stochastic Dynamic Programming, using the computer program developed by Turgeon (2005). The lag-one autocorrelation of the inflows was taken into account, which means that the conditional probability $\Pr\left(Q_t \middle| Q_{t-1}\right)$ was also used to solve the optimization problem. The optimal operating policy was determined by solving an equation similar to (4-1), but for $N = 11$ only. In other words, rather than using 100, 200 or 300 values of $Q_t$ for solving the optimization problem, Turgeon (2005) used only 11 values. These values were:

$$\text{(the mean)} + \lambda_i \text{ (the standard deviation)}$$

where $\lambda_i$ = -2.5, -1.82, -1.36, -0.91, -0.45, 0.0, 0.45, 0.91, 1.36, 1.82, 2.5. The results obtained by Turgeon (2005) are given in Table 2. Strangely, they are better than the results presented in Table 1 which were obtained with Sampling Stochastic Dynamic Programming. There may be several reasons for this. First, the computer program used by Turgeon (2005) and that used in this thesis may be different. Second, synthetic inflows generated with a linear autoregressive model rarely have the same statistics than the original data. For instance, synthetic inflows often have greater extreme values than the original data. This may of course have an effect on the operating policy. The only conclusion that we may draw from comparing results of Table 1 and Table 2 is that there does not seem to be any advantage in using Sampling Stochastic Programming instead of Stochastic Dynamic Programming for solving stochastic optimization problems.

**Table 4-6 Results obtained with the method used by Turgeon (2005)**

| The number of simulation | Mean annual value of | |
|---|---|---|
| | benefit | penalty |
| M=100 | 415981 | 0 |
| M=200 | 414103 | 0 |
| M=300 | 412427 | 0 |

## 4.5 Computer program

A visual C++ code was written for solving the optimization problem solved with SSDP. This computer program consists of ten functions or sub-routines named:

- main
- aleas
- lect_installation
- lect_apport
- distribution_probabilite
- correlogramme
- regression
- scenario_apports
- optimisation
- simulation

The **main** function reads the parameters of the simulation and calls all the other functions in the correct order. The **aleas** function generates a set of random numbers that are normally distributed with a mean of zero and a variance equal to 1. The function named **lect_installation** reads the characteristics of the installations as well as the minimum and maximum constraints on the reservoir level and discharge. These constraints vary throughout the year. The function **lect_apport** reads the historical inflow data. The function named **distribution_probabilite** determines the probability distribution of the inflows in each day of the year. When the inflows in a given day are not normally distributed, the function transforms the inflow data into data that are normally distributed using the lognormal distribution with 2 or 3 parameters or the Gamma distribution. The choice of the distribution is made with the Filliben test. The function named **correlogramme** determines the correlation between the inflows in day $k$ and those in days $k-1, k-2, \ldots, k-\text{no\_lags}$, where no_lags is a parameter whose value is set by the user and read by the **main** function. The function **regression** determines the values of the coefficients in the autoregressive model of the daily inflow. The function named **scenario_apports** generates a fixed number of daily inflow scenarios using the autoregressive model built with **regression**. The number of scenarios is a parameter set by the user and read by the **main** function. Each scenario is a year long and has therefore 365 values. The most important function is certainly **optimization** because it determines the optimal operating policy of the reservoir for the 365 days of the year by solving functional equation (4-1) backward in time. Finally, the function **simulation** simulates the operation of the reservoir over a period of $N$ years using the operating policy determined by **optimization**, and more specifically using the functions $f_t\left(S_t, Q_{i,t-1}\right)$ determined by (4-5). The number of years of simulation $N$ is a parameter set by the user and read by the **main** function.

**CONCLUSION**

In this thesis, the method of sampling stochastic dynamic programming was applied to case study (Quebec).In method of sampling Stochastic Dynamic programming, two options for transition probability were applied and it was shown that the performance of method of SSDP is significantly affected by choosing suitable probability.

We come also to the conclusion that there are no real advantages in using Sampling Stochastic Dynamic Programming instead Stochastic Dynamic Programming to determine the operating policy of a reservoir.

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